cone theory‡ in the region upstream of the trip strip and with turbulent theory downstream. (schlieren photographs show that turbulent flow existed behind the trip strip.) The results for the various control surfaces can be summarized as follows: 1) the 17.5° kink model shows an increase in centerline heat transfer of 3.4 times the straight cone value; 2) the 25° kink model shows an increase of 3.75 times the straight cone value; and 3) the air brake tab model shows a factor of 5. There is also good agreement between these values and those generated from $(h_{\rm cone}/h_{\rm tab}) = (p_{\rm cone}/p_{\rm tab})^{4/5}$ with the pressures based upon theoretical values from the cone tables.

References

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‡ Eckert's reference temperature method.

Preliminary Results on Boundary-Layer Stability on a Flexible Plate

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THE effect of flexible walls on the stability of plane Poiseuille flow was presented in Ref. 1. The same numerical procedure was used to solve the stability of the laminar boundary layer on a flat plate whose surface is flexible. This note contains stability diagrams for the Tollmein-Schlichting mode modified by flexibility and preliminary results on the other modes.

The wall is a membrane of mass M stretched with tension T, and has a damping coefficient D. The flexible surface is assumed to remain in the position of a rigid wall until a small disturbance is introduced into the laminar boundary layer. The resultant pressure fluctuations produce oscillations of the wall which in turn alter the stability of the flow. Because the wall is now in motion, several other modes of instability appear in addition to the Tollmein-Schlichting waves.

The Orr-Sommerfeld equation was solved numerically for the eigenvalue c by the procedure outlined in Ref. 2 for each value of the wave number α and Reynolds number R. The boundary conditions specifying no slip at the wall are:

$$\zeta(d^3\phi/dy^3) + \phi = 0 \tag{1}$$

$$c(d\phi/dy) = -2.6564\phi \tag{2}$$

where

$$\zeta^{-1} = \left[K_2R + \frac{2.6564}{c}\right]\alpha^2 - i\left[\frac{K_3\alpha^3}{c} - m\alpha^3c\right] \tag{3}$$

In addition,

$$\left.\begin{aligned} K_2 &= D/\rho_\infty u_\infty & K_3 &= T/\mu_\infty u_\infty \\ m &= Mu_\infty/\mu_\infty & R &= \rho_\infty u_\infty \delta/\mu_\infty \end{aligned}\right\} \tag{4}$$

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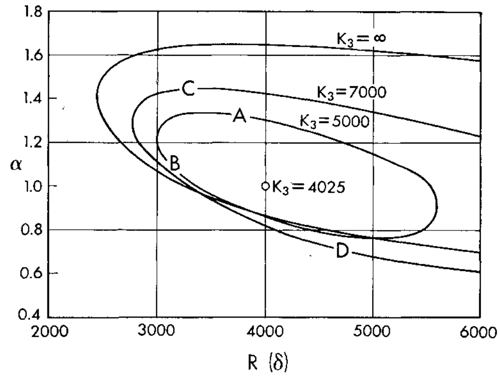

Fig. 1 Neutral stability curves for disturbances of wavelength $2\pi/\alpha$ as a function of Reynolds number for various values of wall tension. Blasius velocity profile on a flexible wall with $K_2 = m = 0$.

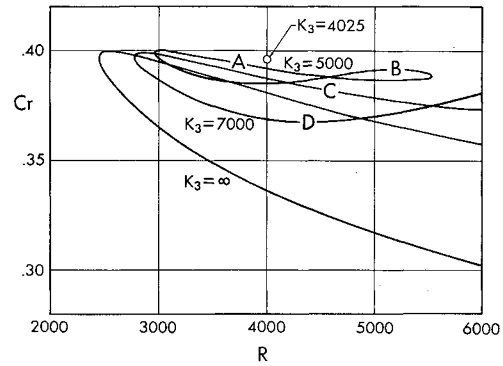

Fig. 2 Neutral stability curves for the wave propagation velocity c_r of disturbances as a function of Reynolds number for various values of wall tension. Blasius velocity profile on a flexible wall with $K_2 = m = 0$.

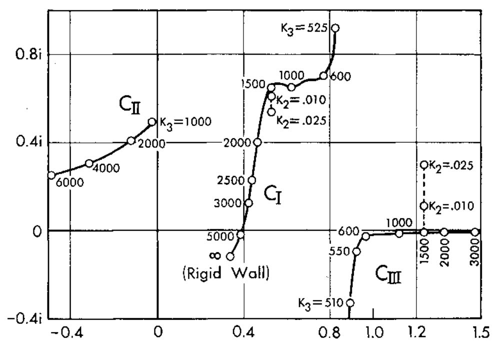

Fig. 3 Eigenvalues as a function of wall tension and damping. $\alpha = 1$, $R = 5000$, $m = 0$, and, unless specified otherwise, $K_2 = 0$.

where ρ_∞, u_∞, and μ_∞ are the freestream density, velocity, and viscosity, respectively. All lengths have been nondimensionalized with respect to the boundary-layer thickness δ.

The neutral stability curves for the wave number and the propagation speed are shown in Figs. 1 and 2, respectively, for the Tollmein-Schlichting waves over walls with increasing flexibility. The quantity K_3 has been nondimensionalized without the use of the thickness δ so that the change in Reynolds number must be interpreted as a change in δ only. The stability curves form closed loops and shrink until they disappear at a finite value of K_3. The curves probably do not form closed loops, but only shift to higher R if they are cross-plotted to allow change in R by variation of one of the other quantities such as u_∞. Preliminary results using double precision on the IBM 7094 show the appearance of instabilities for Tollmein-Schlichting waves with longer wavelengths when $\alpha < 0.1$. Further calculations in the low Reynolds number and low α region are required.

The effect of wall tension on the three modes of instability for a particular wavelength of the disturbance and Reynolds number is shown in Fig. 3. The disturbance is stable when the imaginary part of c is positive and unstable when it is negative. C_{I} is the Tollmein-Schlichting mode modified by flexibility. C_{II} is a stable wave which propagates in the upstream direction. The third eigenvalue C_{III} is unstable and the eigenfunction becomes unbounded as $c \rightarrow 1$. This can usually be cured by addition of wall damping as shown for $K_3 = 1500$. Damping is destabilizing for C_{I}, but to a lesser extent, so that it remains stable Additional calculations are required to obtain neutral stability curves for C_{II} and C_{III} as well as for all the eigenvalues in the region where $\alpha < 0.1$. These results will be presented in the near future.

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Design of Rectangular Panels with Biaxial Stresses

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Nomenclature

a = length of rectangular panel
b = width of rectangular panel
C_{mn} = amplitude of deflection of mn mode
m = number of half-waves in longitudinal (x) direction
n = number of half-waves in transverse (y) direction
t = panel thickness
w = lateral deflection
x = longitudinal direction
y = transverse direction
C_τ = stability constant for panel in pure shear
D = bending stiffness of panel $[Et^3/12(1 - \nu^2)]$
E = effective modulus of elasticity
K_c = equivalent stability constant for biaxial compression
N_x = compression loading per inch acting in x direction
N_y = compression loading per inch acting in y direction
N_{xy} = shear loading per inch
$\bar{N}_x$ = equivalent uniaxial compression loading in x direction for a panel loaded in biaxial compression and shear
γ = ratio of compression loadings (N_y/N_x)
ξ = nondimensional parameter (m^2b^2/n^2a^2)
σ_x = stress in x direction
σ_y = stress in y direction
$\bar{\sigma}$ = characteristic stress function (π^2D/b^2t)

The analysis of a simply supported rectangular panel subjected to biaxial-compression is treated in paragraph 64 of Ref. 1. The analysis determines the axial stress σ_y that exists when the panel buckles because of a known axial stress σ_x. The formulation is not convenient, however, for designing a panel when only the loads (N_x and N_y) are known. The usual interaction type of equation, which treats the over-all stability as functions of the stability of independent modes, can only be an approximation since the buckle pattern will change with the aspect and load ratios. The technique recommended is to use the standard design technique for a uniaxially loaded plate with an equivalent stability constant that is a function of the aspect and load ratios.

The equilibrium equation for the rectangular plate with biaxial loading is

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} = 0 \quad (1)$$

Substituting the general eigenvector

$$w = C_{mn}\sin(m\pi x/a)\sin(n\pi y/b) \quad (2)$$

which satisfies the simply supported boundary conditions, into Eq. (1) results in the following relationship which is satisfied by the critical loading:

$$N_x\left(\frac{m^2\pi^2}{a^2}\right) + N_y\left(\frac{n^2\pi^2}{b^2}\right) = D\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)^2 \quad (3a)$$

Let

$$\bar{\sigma} = \pi^2 D/b^2 t \quad (3b)$$

$$\xi = m^2b^2/n^2a^2 \quad (3c)$$

$$a > b \qquad N_x/t = \sigma_x > \sigma_y = N_y/t \quad \text{(compression positive)} \quad (3d)$$

$$\gamma = N_y/N_x = \sigma_y/\sigma_x \leq 1 \quad \text{(can be negative)} \quad (3e)$$

Substitute the preceding in Eq. (3a) to determine the minimum value of σ_x satisfying the buckling condition. This results in

$$\sigma_x = \min[\bar{\sigma}n^2(1 + \xi)^2/(\gamma + \xi)] \quad (3f)$$

where m and n are integers.

The minimum value of σ_x corresponds to $n = 1$ (σ_x is monotonic with n) and for the smallest integer value of m which is greater than $(a/b)(1 - 2\gamma)^{1/2}$

The equation

$$m \geq (a/b)(1 - 2\gamma)^{1/2} \qquad (\gamma \leq \tfrac{1}{2}) \quad (4a)$$

is obtained by setting $\partial\sigma_x/\partial\xi = 0$ and solving for m when $n = 1$. For values of γ greater than $\frac{1}{2}$, the value of σ_x is monotonic with respect to ξ and the minimum value occurs at the smallest value of ξ. This corresponds to

$$m = 1 \qquad (\gamma \geq \tfrac{1}{2}) \quad (4b)$$

The design of a simply supported panel can be obtained, therefore, as follows:

1) For the given aspect ratio $a/b \geq 1$ and load ratio $N_y/N_x = \gamma \leq 1$, determine the smallest integer $m \geq (a/b)(1 - 2\gamma)^{1/2}$ (for $\gamma > \frac{1}{2}$, $m = 1$).
2) Determine $\xi = m^2b^2/a^2$.
3) Establish equivalent uniaxial stress equation

$$\sigma_x = \frac{(1 + \xi)^2}{\gamma + \xi}\frac{\pi^2}{12(1 - \nu^2)}E\left(\frac{t}{b}\right)^2 = K_cE\left(\frac{t}{b}\right)^2 \quad (5a)$$

with

$$K_c = \frac{(1 + \xi)^2}{\gamma + \xi}\frac{\pi^2}{12(1 - \nu^2)} \quad (5b)$$

4) Design plate with K_c rather than $[(mb/a) + (a/mb)]^2[\pi^2/12(1 - \nu^2)] \rightarrow 3.62$ which corresponds to the uniaxial compression case. For the elastic case the design equation becomes

$$t = (N_xb^2/K_cE)^{1/3} \quad (6)$$

For plastic design an effective modulus must be defined and the plate designed by a graphical method (e.g., see subsection IIC 1 of Ref. 2).

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